JC-003-1016003 Seat No. ______

B. Sc. (Sem. VI) (CBCS) Examination

August - 2019

Mathematics : Paper - 10 (A)

(Optimization & Numerical Analysis-II)

Faculty Code : 003

Subject Code : 1016003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Figure to the right indicate full marks of the questions.

1 (a) Answer the following questions : **4**

(1) Define : Surplus variable.

(2) Which method is suitable to find optimum solution of any LPP that contains some of the constraints of the type greater than or equal ?

(3) If $C_j - Z_j = 0$ corresponds to one of the non basic variables and $C_j - Z_j < 0$ correspond to rest non basic variables appear in simplex algorithm, then the solution of given LPP is of the type alternate optimum. (True / False)

(4) Define : Simplex.

(b) Attempt any **one** : **2**

(1) State an example of LPP and express it into standard form.

(2) Define :

(i) Feasible solution.

(ii) Convex set.

(c) Attempt any **one** : **3**

(1) Write short note on types of solutions of LPP.

(2) Solve the following LPP using graphical method :

Max. $Z = -2x_1 - x_2$, subject to the constraints :

$3x_1 + x_2 = 3,$
$4x_1 + 3x_2 \geq 6,$
$x_1 + 2x_2 \leq 4$
and $x_1, x_2 \geq 0.$

(d) Attempt any **one** : **5**

(1) Maximize $Z = 2x_1 + 4x_2 + x_3 + x_4$; subject to :

$2x_1 + x_2 + 2x_3 + 3x_4 \leq 12,$
$3x_1 + 2x_3 + 2x_4 \leq 20,$
$2x_1 + x_2 + 4x_3 \leq 16$
and $x_1, x_2, x_3, x_4 \geq 0.$

(2) Write algorithm for big-M method.

2 (a) Answer the following questions : **4**

(1) If the matrix form of LPP is : maximize $Z = CX$, subject to $AX \leq B, X \geq 0$, then dual of the primal is ________.

(2) If a transportation problem of n supply points and m demand points has non-degenerate solution, then the number of allocation in basic feasible solution is ________.

(3) Write name of method, that optimizes solution of transportation problem.

(4) Write name of method, to find optimum solution of assignment problem.

(b) Attempt any **one** : **2**

(1) Write dual of the LPP : Minimize $Z = 2x_2 + 5x_3$, subject to

$x_1 + x_2 \geq 2,$
$2x_1 + x_2 + 6x_3 \leq 6,$
$x_1 - x_2 + 3x_3 = 4$
and $x_1, x_2, x_3 \geq 0.$

(2) Explain North-West corner method in brief.

(c) Attempt any **one** : **3**

(1) Write a note on primal-dual relationship.

(2) Solve the following transportation problem by using Least Cost Entry Method :

Warehouse → Factory ↓	W_1	W_2	W_3	W_4	Factory Capacity
F_1	40	8	70	20	18
F_2	70	30	40	60	9
F_3	19	30	50	10	7
Warehouse Requirement	5	8	7	14	34

(d) Attempt any **one** : **5**

(1) Describe Hungarian method.

(2) Find optimum solution of the following transportation problem :

		Destination					
		d_1	d_2	d_3	d_4	d_5	supply
Source	Delhi	4	2	3	2	6	8
	Surat	5	4	5	2	1	12
	Pune	6	5	4	7	3	14
	Demand	4	4	6	8	8	

3 (a) Answer the following questions : **4**

(1) Which interpolation formula is an outcome of average of Gauss-Forward interpolation and Gauss-Backward interpolation formula ?

(2) Newton's divided difference interpolation is same as _________ if domain data is at equidistance.

(3) If the value of $p = \frac{x - x_0}{h}$ lies in $\left[\frac{3}{4}, 1\right]$, the best suitable interpolation to apply is _________.

(4) Lagrange interpolation formula is applicable only when the arguments are not at equidistance. (True/False)

(b) Attempt any **one** : **2**

(1) If $f(x) = x^{-1}$, prove that the third order divided difference $f(1, 2, 3, 4) = \frac{1}{4!}$.

(2) State Lagrange's interpolation formula.

(c) Attempt any **one** : **3**

(1) Derive Newtons divided interpolation formula.

(2) Apply Stirling's formula to find a polynomial which takes the values as per the following table :

x	1	2	3	4	5
y	1	−1	1	−1	1

(d) Attempt any **one** : **5**

(1) Derive Laplace-Everett's interpolation formula.

(2) Using following table, find the value of x at which the value of $y = 13.6$.

x	30	35	40	45	50
$f(x)$	15.9	14.9	14.1	13.3	12.5

4 (a) Answer the following questions : **4**

(1) Write first derivative of Gauss-Forward interpolation formula.

(2) Minimum number of subintervals required to apply Trapezoidal and Simpson's 3/8 rule simultaneously is _________.

(3) State error formula of Simpson's $\frac{1}{3}^{\text{rd}}$ rule.

(4) State Simpson's 3/8 rule.

(b) Attempt any **one** : **2**

(1) Derive second order derivative of Newtons-Forward interpolation formula.

(2) By using Simpson's $\frac{1}{3}^{\text{rd}}$ rule, find $\int_0^1 y\, dx$ from the following observations :

x	0	0.25	0.5	0.75	1
y	1.0000	1.2840	1.6487	2.1170	2.7183

(c) Attempt any **one** : **3**

(1) Derive first and second order derivative formula from Bessel's interpolation formula.

(2) Estimate the length of the arc of the curve $3y = x^3$ from (0, 0) to $\left(1, \frac{1}{3}\right)$ using Simpson's $\frac{1}{3}^{\text{rd}}$ rule taking six sub-intervals.

(d) Attempt any **one** : **5**

(1) Derive Newton-Cotes quadrature formula.

(2) From the following table of values of x and y find y' (1.25) and y'' (1.25) :

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
y	1.00000	1.02470	1.04881	1.07238	1.09544	1.11803	1.14017

5 (a) Answer the following questions : **4**

(1) Runge-Kutta method of second order produces more accurate solution than improved Euler method. (True/False)

(2) Which type of differential equation can be solved by Runge's method ?

(3) Write name of any one predictor-corrector method.

(4) Write formula of modified Euler's method.

(b) Attempt any **one** : **2**

(1) Give geometric interpretation of improved Euler method to solve differential equation.

(2) Write working rule of Runge's method to solve differential equation.

(c) Attempt any **one** : **3**

(1) Explain Picard's method to solve simultaneous differential equation.

(2) Using Runge-Kutta method of fourth order, find $y(1.1)$, $y(1.2)$, given that $2y' = 2x^3 + y, y(1) = 2$.

(d) Attempt any **one** : **5**

(1) Explain Milne's method.

(2) Employ Taylor's method to obtain approximate value of y at $x = 1.2$ for the differential equation $\frac{dy}{dx} = 2y + 3e^x$, given that $y(1) = 0$. Compare the numerical solution with the exact solution.
